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Collateralizing Ideas: Intangibles in the Credit Market

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Abstract

Intangible capital comprises an increasing share of total capital assets, and its non-physical nature makes it more difficult to evaluate and secure as collateral for loans. I extend the model of intangible capital presented in McGrattan and Prescott (2010) to include a collateralized credit market in which firms can obtain debt proportional to their capital assets. I consider different cases for the relative collateral value of intangibles under a credit constraint subject to exogenous shocks. For greater collateralizability of intangible assets, the model predicts a stronger negative relationship between intangible investment and credit availability and more stable interest rates. However, the model overall does not replicate observations from macroeconomic data.

1 Introduction

With the shift of modern economies towards services and digital goods instead of traditional heavy industry, firms require new types of capital to meet the demands of the market. Large technology firms, arguably some of the biggest and most powerful producers in developed nations, do not require sprawling factories or heavy machinery to produce their goods– instead, they rely on R&D, marketing, and software among other inputs. These inputs are often called intangible capital as opposed to physical factors of production like equipment or land. Although intangible capital behaves similarly to tangible capital in many ways, its non-physical nature and often nebulous role in a product's value makes it challenging to account for. The value of a factory, for instance, is not difficult to estimate, whereas the value added to an item because of the marketing and branding efforts of the firm is less clear. Regardless of the accounting challenges, intangibles undoubtedly comprise a significant portion of modern capital and investment: Elsten and Hill (2017) estimate that 85% of the S&P 500 companies' value is in intangibles, and Bontadini et al. (2021) find that intangible investment has outgrown tangible investment for over a decade. As of 2021, intangible investment stands at over 16% of private GDP compared to just over 8% for tangible investment (Bontadini et al., 2021).

Van Ark et al. (2009) suggest three general categories of intangible capital: computerized information such as software and databases, innovative property including scientific and non-scientific R&D, and economic competencies such as branding, human resources, and organizational structures. While records of intangibles have improved over time, many subsets such as the development of financial products and economic competencies are still omitted from measures of GDP (Bontadini et al., 2021). In contrast to tangible capital, intangibles are often considered non-rival (Crouzet et al., 2022) as assets like brand reputation or organizational efficiency can be used across the firm without depletion. Intangibles may also be less excludable than tangibles as gains to scientific knowledge and human capital can spread outside of the firm (Crouzet et al., 2022). These traits contribute to the 'spillover' effects wherein intangible investment not only increases the intangible capital stock, but also contributes to productivity growth even beyond R&D expenditures (Van Ark et al., 2009; Corrado, Haskel, and Jona-Lasinio, 2017). Previous work has found positive relationships between R&D and innovative patents and TFP (Dancaková et al., 2022), as well as for human capital investment (Habib, Abbas, and Noman, 2019; Kubin and Zörner, 2021) and investment in the relationship between firms and customers (Gourio and Rudanko, 2014). Despite the persistently low reporting of even the most widely recognized forms of intangible capital such as R&D (Dancaková et al., 2022), intangibles emerge as key factors in driving the economic and technological growth of modern economies.

Consideration of intangibles is also often necessary to explain the behaviour of macroeconomic quantities. McGrattan and Prescott (2010) find that accounting for intangibles allows real business cycle models to reconcile the seemingly paradoxical low wages and high output of the 1990s. Similarly, Mitra (2019) reports that the greater share of intangibles in the economy accounts for the less procyclical behaviour in labour productivity of recent years and reduced output volatility, and Gourio and Rudanko (2014) find the volatility of the labour wedge results from flawed measurements in intangible capital. Other models suggest that intangibles are key to propagating shocks through the aggregate economy, where the productivity of the intangible sector has lasting impacts on output (Malik, Ali, and Khalid, 2014).

The rising importance of intangibles coupled with the challenges associated with measuring their worth creates many questions as to their role in production and other activities of the firm. In particular, I focus on the role of intangibles in the collateral constraint for securing debt. Pledging collateral to obtain debt reduces the moral hazard problem and allows debtors to borrow more by alleviating some of the costs of screening to creditors (Bernanke, 2018). However, with firms spending more resources on producing oft-unmeasured intangibles, their apparent book value may not reflect the total value of their assets and the amount they can offer as collateral may suffer. Previous literature has found credit and investment to be positively related, though the causality is unclear (Rioja, Rios-Avila, and Valev, 2014): investment may boost credit by creating additional assets to borrow against, or credit may enable greater investment by supplying firms with additional resources. Models such as that presented by Monacelli (2009) find that including collateral constraints improves the fit between model behaviour and observations in response to certain shocks, while others have found that such financing helps to propagate uncertainty shocks through the economy (Gourio, 2013). Growth is also affected by the availability of credit, and models predict lower growth when borrowing is more constrained (Azariadis and Kaas, 2016).

The collateral constraint does not allow borrowers to obtain debt equal to the full value of their assets, as there are costs to lenders associated with screening and foreclosure that reduce the retrievable worth of the assets. Bernanke (2018) reports that the costs of external finance are countercyclical as debtor net worth drops in economic recessions and endogenous constraints tighten in response to negative shocks. Tighter credit constraints induce more procyclic behaviour in long-term investments as well by making long-duration investments appear more risky and thus more undesirable during economic downturns. This channel between credit and investment amplifies the procyclicality of economic growth (Aghion et al., 2010). The tightening of financial constraints exacerbates the effects of economic downturns, though Ai et al. (2020) report that greater asset collateralizability helps insure against shocks by reducing the riskiness of lending to firms.

Additionally, shocks to collateral value can interfere with capital allocation and generate or accentuate productivity shocks (Azariadis and Kaas, 2016). Buera and Moll (2015) find that unemployment dynamics are driven by credit crunches more so than TFP fluctuations, in addition to the reallocation of resources away from younger, relatively more productive, and more financially constrained firms towards older and relatively unproductive firms. Likewise, the productivity of newer sectors of the economy that face stricter financial constraints is more sensitive to credit shocks, and the resulting resource misallocation interferes with structural change in the economy as a whole (Hirakata and Sunakawa, 2019). Ai et al. (2020) find that firms in greater need of financing endogenously accumulate collateralizable assets, and together, this implies that firms' decisions may depend on their need for external credit. The combined uncertainties of future financial constraints and returns to investment prompt firms to save more instead of spending in the current period (Amable, Chatelain, and Ralf, 2010).

The function served by intangible capital in collateral constraints is less than clear. Some previous work has assumed that intangible investment cannot be externally financed at all: Corrado, Haskel, and Jona-Lasinio (2019) present a model where firms must save and self-finance in order to invest in intangibles. In this regime, intangible investment is suppressed by low interest rates when saving is less lucrative. This is supported by empirical evidence that intangible investment is less sensitive to banking-related variables (Corrado, Haskel, and Jona-Lasinio, 2019) and that firms with greater R&D investment tend to have lower debt as compared to firms with larger physical investment (Wang, 2017). The problem of asymmetric information plagues all credit markets and is worse for intangibles such as R&D where projects may be poorly recorded and difficult to predict (Amable, Chatelain, and Ralf, 2010). However, the literature also suggests that intangibles can be made collateralizable, particularly through patenting. Horsch, Longoni, and Oesch (2021) find that invalidating a firm's patents reduces its leverage and report a causal positive effect of intangible capital on leverage. In fact, the importance of considering the role of intangibles in debt has been recognized as early as in Klock, Baum, and Thies (1996), where the inclusion of intangibles improves the performance of the model of firms' investment decisions.

Collateralizing intangibles via patents has received extensive attention, as patents serve the dual functions of providing a collectable asset to pledge as collateral while also improving records of intangibles (Hochberg, Serrano, and Ziedonis, 2014). In general, undeclared assets cannot be pledged for loans and so firms with larger quantities of such hidden assets face tighter credit constraints (Granda Carvajal, 2015). Hottenrott, Hall, and Czarnitzki (2015) find that both the patent stock of a firm and its recent patent acquisitions reduce financing constraints in well-established firms; furthermore, their findings imply that patents could provide a 'quality signal' for the innovative productivity and managerial efficiency of the firm which improves lender confidence. Thus, intangibles may reduce the debt available to firms not because they cannot be collateralized, but because they are poorly documented. A more liquid patent market for the resale of patents after foreclosure reduces financial frictions and improves access to debt for startup firms (Hochberg, Serrano, and Ziedonis, 2014) and enforcement of patents and similar intellectual property rights boosts the human capital and R&D per worker (Habib, Abbas, and Noman, 2019).

Even under credit constraints limited by patents, Amable, Chatelain, and Ralf (2010) find the rate of return on innovation to outstrip the interest rate on the loans in a growing economy. High human capital has also been noted to mitigate instability arising from financial frictions (Kubin and Zörner, 2021). A feedback loop emerges between intangible investment into human capital and credit: higher human capital reduces the screening costs of loans and contributes to credit market development (Ho, 2013). Lopez and Olivella (2018) find that including intangibles in a credit-constrained model is necessary to reproduce the observed volatility of output and employment, and similarly, Pastorino, Kehoe, and Midrigan (2016) also find intangible accumulation to be key in amplifying employment shocks and the impacts of credit frictions. The inclusion of intangible assets allows borrowers to smooth consumption when credit becomes less available (Lopez and Olivella, 2018) and overall increases welfare, labour demand, and investment in both physical and intangible capital (Wang, 2017).

I formulate a model of intangible capital and investment similar to the extended model of McGrattan and Prescott (2010). Rather than focus on patents or other subsets of intangibles, I consider the impact of exogenous credit shocks on firms' resource allocation towards output or intangible capital and vary the parameter controlling the collateral value of intangible capital to explore its role in the credit constraint. McGrattan and Prescott (2010) present a model for intangible investment in which the inputs to production, capital and labour, are also used to produce intangibles. They find that the discrepancy between standard model predictions and measured output through the 1990s can be resolved when intangibles are accounted for and productivities are allowed to vary between the output and intangible sectors (i.e. non-neutral technology). Specifically, the wage rate is determined by the marginal productivity of labour in the output sector despite labour also being supplied to the intangible sector, leading to seemingly low wages when intangible investment is not considered. The large amount of technological R&D and similar investment into intangibles creates a boom in intangible productivity, again not reflected in traditional models and accounts.

I then extend the McGrattan and Prescott (2010) model by allowing agents to borrow up to some fraction of their total assets the collateral limit as in Iacoviello (2005) by introducing impatient entrepreneurs who borrow in real terms from patient households. Iacoviello (2005) specifically models nominal debt with housing tied to housing stock, in which the inclusion of collateral enables the model to replicate the response of consumption to shifts in housing prices. The analog in my model appears as movements in consumption when credit expands and assets can be more liberally pledged as collateral. However, I break from Iacoviello (2005) and let interest rates be determined by an optimal investment rule instead of a central bank which couples the productivity of capital to the cost of debt. I also allow the credit constraint to vary exogenously similar to the 'credit crunch' formulation of Buera, Jaef, and Shin (2015), where a credit shock appears as an unexpected change in the ratio of loans to capital stock. Buera, Jaef, and Shin (2015) find that a negative credit shock, or a constricting of the collateral constraint, induces a decline in output, capital stock, and employment. I find similar results for the responses of output and labour but the reverse for capital stock due to the effects of the optimal investment rule.

The inclusion of debt and the collateral constraint introduces much greater variance into the model. As previously discussed, credit frictions and loan market dynamics amplify and propagate shocks through the economy. This produces unreasonable volatility in several variables, which I mitigate by including habit persistence in the household's preferences and adding a durable wealth stock akin to housing in Iacoviello (2005) that serves to stabilize the wealth of agents across periods.

2 Model without borrowing

I begin with the model containing intangible investment and non-neutral technology similar to the model presented in McGrattan and Prescott (2010), which features intangible capital as a factor of production and distinct paths for productivity growth for the production of tangible and intangible goods.

2.1 The household

The representative household owns the tangible $k_{T,t}$ and intangible $k_{I,t}$ capital and rents it to the firm for rates $r_{T,t}$ and $r_{I,t}$, respectively. It provides labour-hours h_t for wages w_t, and each period, the household must choose between consumption c_t and capital investment $x_{T,t}$ and $x_{I,t}$. q_t denotes the relative price of intangible investment.

The household also must pay taxes on its activities: τ_c on consumption, τ_h on labour income, τ_k on tangible capital, τ_p on profits, and finally τ_d on capital distributions. I assume these rates are constant and that the household finances fraction χ of intangible investment– that is, $\chi q_t x_{I,t}$ is expensed from accounting records. All together, the household maximizes utility given by:

$$E\sum_{t=0}^{\infty}\beta^t\left[\log(c_t)+\psi\log(1-h_t)\right] \tag{2.1}$$

subject to

$$\begin{aligned} c_t + x_{T,t} + q_t x_{I,t} = {} & r_{T,t}k_{T,t} + r_{I,t}k_{I,t} + w_t h_t \\ & - \tau_c c_t - \tau_h(w_t h_t - (1-\chi)q_t x_{I,t}) - \tau_k k_{T,t} \\ & - \tau_p(r_{T,t}k_{T,t} + r_{I,t}k_{I,t} - \delta_T k_{T,t} - \chi q_t x_{I,t} - \tau_k k_{T,t} \\ & - \tau_d\left[r_{T,t}k_{T,t} + r_{I,t}k_{I,t} - x_{T,t} - \chi q_t x_{I,t} - \tau_k k_{T,t}\right. \\ & \quad \left. -\tau_p(r_{T,t}k_{T,t} + r_{I,t}k_{I,t} - \delta_T k_{T,t} - \chi q_t x_{I,t} - \tau_k k_{T,t})\right] \end{aligned} \tag{2.2}$$

$$k_{T,t+1} = (1-\delta_T)k_{T,t} + x_{T,t} \tag{2.3}$$

$$k_{I,t+1} = (1-\delta_I)k_{I,t} + x_{I,t} \tag{2.4}$$

where (2.2) is the household's budget constraint and (2.3) and (2.3) are the laws of motion for capital. δ_T and δ_I are the depreciation rates for tangible and intangible capital, respectively.

2.2 The firm

Firms use inputs $k_{T,t}, k_{I,t}$ and h_t to produce output y_t and intangible investment $x_{I,t}$. As in McGrattan and Prescott (2010), I assume that tangible capital and labour must be divided between the sectors, whereas intangible capital is non-rival and can be used fully in both production functions. Intuitively, this arises from how many subsets of intangible capital, such as organizational competence, technological prowess, and branding, are not depleted by use and can be employed simultaneously across different branches of the firm. The constant returns-to-scale production functions for output and intangible investment are as follows:

$$y_t = A_t^1(k_{T,t}^1)^\theta(k_{I,t})^\phi(h_t^1)^\alpha \tag{2.5}$$

$$x_{I,t} = A_t^2(k_{T,t}^2)^\theta(k_{I,t})^\phi(h_t^2)^\alpha \tag{2.6}$$

where the aggregations for labour and tangible capital, $h_t = h_t^1 + h_t^2$ and $k_{T,t} = k_{T,t}^1 + k_{T,t}^2$, must hold. I follow McGrattan and Prescott (2010) and assume equal productivity of inputs in both sectors. Total factor productivities for output A_t^1 and intangible investment A_t^2 follow AR(1) processes subject to exogenous shocks. The firm's profit-maximization problem yields the wage and rental rates:

$$w_t = \alpha \frac{y_t}{h_t^1} \tag{2.7}$$

$$r_{T,t} = \theta \frac{y_t}{k_{T,t}^1} \tag{2.8}$$

$$r_{I,t} = \phi \frac{y_t}{k_{I,t}} \tag{2.9}$$

(2.7), (2.8), and (2.9) assume that wages and rental rates are determined by the marginal productivity of inputs in the output sector alone without consideration for the intangible sector. Following McGrattan and Prescott (2010), the shares of tangible capital and labour allocated to production are assumed to obey:

$$h_t^1 = \frac{y_t}{q_t x_{I,t}} h_t^2 \tag{2.10}$$

$$k_{T,t}^1 = \frac{y_t}{y_t + q_t x_{I,t}} k_{T,t} \tag{2.11}$$

where (2.10) and (2.11) are derived from equating the marginal products of each input across the sectors, and h_t^2 and k_t^2 are determined from the residuals. Finally, the economy's aggregate feasibility constraint is given by:

$$y_t = c_t + x_{T,t} + x_{I,t} + g_t \tag{2.12}$$

where g_t is government expenditure and is residually determined.

I then log-linearize this model according to the rules presented in Uhlig (1999) and estimate the steady-state values of output, consumption, and tangible and intangible investment using data from 2007 through 2017.

3 Extended model with collateral-constrained borrowing

To incorporate borrowing, I adapt the collateral framework as in the basic model presented in Iacoviello (2005) and introduce new agents, the patient household and the impatient entrepreneur.

3.1 The household

As before, the household consumes c_t and provides labour h_t; however, the household also can now lend or borrow amount b_t at interest rate i_t. I also include durable 'housing' stock d_t in the household's problem: this provides a consistent source of wealth for the household and mitigates excess volatility in the household's decisions. Without d_t, the household's only wealth comes from labour and lending income and generates exceedingly high variance in consumption. I also assume consumption habit persistence for the household given by parameter p. The household thus chooses the set of values $\{c_t, h_t, b_t, d_t\}$ to maximize utility:

$$E\sum_{t=0}^{\infty}\beta^t\left[\log(c_t - pc_{t-1}) + \psi\log(1-h_t) + \nu\log(d_t)\right] \tag{3.1}$$

subject to the budget constraint

$$c_t + b_{t-1}(1+i_{t-1}) + q_{d,t}(d_t - d_{t-1}) = w_t h_t + b_t - \tau_c c_t - \tau_h(w_t h_t) \tag{3.2}$$

where the household pays taxes on consumption and its labour income. From the household's problem, I obtain the following first-order conditions:

$$\frac{\beta p}{(c_{t+1} - pc_t)(1-\tau_c)} = \frac{\psi}{(1-h_t)w_t(1+\tau_h)} + \frac{1}{(c_t - pc_{t-1})(1+\tau_c)} \tag{3.3}$$

$$\frac{\beta(1+i_t)}{(1-h_{t+1})w_{t+1}} = \frac{1}{(1-h_t)w_t} \tag{3.4}$$

$$\frac{\beta q_{d,t+1}}{(1-h_{t+1})w_{t+1}(1-\tau_h)} = \frac{q_{d,t}}{(1-h_t)w_t(1-\tau_h)} + \frac{\nu}{d_t} \tag{3.5}$$

where (3.3) equates the marginal utilities of labour and consumption, (3.4) gives the intertemporal substitution of labour, and (3.5) equates the marginal utilities of labour and durable stock.

3.2 The entrepreneur

Following Iacoviello (2005), I assume that the entrepreneur derives utility from only consumption $c_{e,t}$ and ownership of durable stock $d_{e,t}$ and so works all hours, normalized to 1. The entrepreneur owns the firm and the capital stocks $k_{T,t}$ and $k_{I,t}$, and must pay wages to the working household. The budget constraint and tax structure is similar to that of (2.2) The entrepreneur's utility function is:

$$E\sum_{t=0}^{\infty}\gamma^t\left[\log(c_{e,t})+\nu\log(d_{e,t})\right] \tag{3.6}$$

subject to the budget constraint

$$\begin{aligned} c_{e,t}+x_{T,t}+q_t x_{I,t}+w_t h_t+q_{d,t}(d_{e,t}-d_{e,t-1})+b_{e,t-1}(1+i_{t-1}) = \\ y_t+r_{T,t}k_{T,t}+r_{I,t}k_{I,t}+b_{e,t} \\ -\tau_c c_t-\tau_h(1-(1-\chi)q_t x_{I,t})-\tau_k k_{T,t} \\ -\tau_p(r_{T,t}k_{T,t}+r_{I,t}k_{I,t}-\delta_T k_{T,t}-\chi q_t x_{I,t}-\tau_k k_{T,t} \\ -\tau_d\left[r_{T,t}k_{T,t}+r_{I,t}k_{I,t}-x_{T,t}-\chi q_t x_{I,t}-\tau_k k_{T,t}\right. \\ \left.-\tau_p(r_{T,t}k_{T,t}+r_{I,t}k_{I,t}-\delta_T k_{T,t}-\chi q_t x_{I,t}-\tau_k k_{T,t})\right] \end{aligned} \tag{3.7}$$

and the laws of motion of capital as described in (2.3) and (2.4). As the owner of the capital and the firm, the entrepreneur must pay taxes as in the model without borrowing, again with fraction χ of intangible investment expensed. I assume that the entrepreneur is more impatient than the household so that $\gamma < \beta$, which creates the incentive for entrepreneurs to borrow and ensures that an equilibrium exists when borrowing is equal to the collateral limit. The amount borrowed is limited by the capital stock, and I assume entrepreneurs to borrow up to the maximum allowed:

$$b_{e,t}=m_t\left(\frac{k_{T,t}+nk_{I,t}}{1+i_t}\right) \tag{3.8}$$

where $m_t \in [0,1]$ is the fraction of assets that can be collected as collateral accounting for the transaction costs of repossession, and n is the relative worth of intangible capital as collateral in comparison to tangible capital. Given that intangibles may unaccounted in standard accounting practices or firm-specific, I assume $n \leq 1$ so that intangibles are less than or equally as valuable as collateral than tangibles. While I take n as constant, m_t is exogenous and follows an AR(1) process subject to shocks (Equation 3.9). Negative shocks to m_t serve as 'credit crunches' in which the ratio of loans to assets falls and positive shocks are credit expansions.

$$\log m_t=\rho_m\log(m_{t-1})+\epsilon_m \tag{3.9}$$

I discuss the calibration of ρ_m and ϵ_m below. The interest rate is determined endogenously following an optimal investment rule for tangible capital and output:

$$i_t = \theta \frac{y_{t+1}}{k^1_{T,t+1}} - \delta_T \tag{3.10}$$

where i_t depends only on the marginal productivity and depreciation of tangible capital used in output production. The intuition for this interest rate regime is that the entrepreneur is willing to pay up to the marginal productivity of investment in order to purchase more investment in the current period, as the interest rate acts as the cost of borrowing. I consider only the marginal productivity of tangible capital in output production following the same assumptions as in Equations (2.7) and (2.8) for wage and tangible rental rates.

The functions for production and aggregation of labour and capital are identical to those in the model without borrowing. The loans market must clear so that $b_t = -b_{e,t}$ and lastly, durable goods are assumed to be constant in the aggregate (i.e. $d_t + d_{e,t} = D$) akin to the housing stock in Iacoviello (2005).

The entrepreneur provides the following first-order conditions:

$$\begin{aligned}
\frac{(1-\tau_d)c_{e,t+1}}{c_{e,t}} &= \gamma \Bigg(-\frac{\lambda_{t+1} m_{t+1}(1+\tau_c)}{i_{t+1}+1} + (1-\delta_T)(1-\tau_d) \\
&\quad + .r_{T,t+1} - \tau_d \left(r_{T,t+1} - \tau_k - \tau_p \left(-\delta_T + r_{T,t+1} - \tau_k \right) \right) \\
&\quad - \tau_k - \tau_p \left(-\delta_T + r_{T,t+1} - \tau_k \right) \Bigg)
\end{aligned} \tag{3.11}$$

$$\begin{aligned}
\frac{q_t + \tau_d \chi q_t (\tau_p - 1) - \chi q_t \tau_p - q_t \tau_h (1-\chi)}{c_{e,t}(\tau_c + 1)} &= \gamma \Bigg(-\frac{\lambda_{t+1} m_{t+1} n}{i_{t+1}+1} \\
&\quad - \frac{(1-\delta_I)\left(\chi q_{t+1}\tau_p - q_t + q_{t+1}\tau_h (1-\chi)\right)}{c_{e,t+1}(\tau_c+1)} \\
&\quad - \frac{(1-\delta_I)\left(\tau_d \left(\chi q_{t+1}\tau_p - \chi q_{t+1}\right)\right)}{c_{e,t+1}(\tau_c+1)} \\
&\quad + \frac{\tau_p + r_{I,t+1} - \tau_d \left(-r_{I,t+1}\tau_p + r_{I,t+1}\right) - r_{I,t+1}}{c_{e,t+1}(\tau_c+1)} \Bigg)
\end{aligned} \tag{3.12}$$

$$\gamma \frac{1+i_t}{c_{e,t+1}(\tau_c+1))} = \frac{1}{c_{e,t}(\tau_c+1)} + \lambda_t \tag{3.13}$$

$$\frac{q_{d,t}}{c_{e,t}(\tau_c+1)} = \frac{\nu}{d_{e,t}} + \gamma \frac{q_{d,t+1}}{c_{e,t+1}(\tau_c+1)} \tag{3.14}$$

Equations (3.11) and (3.12) are the first-order conditions with respect to tangible and intangible capital, respectively, while (3.13) gives the intertemporal substitution of entrepreneurial consumption. Lastly, (3.14) provides the substitution between consumption and durable good accumulation.

4 Calibration

Data for aggregate variables were accessed from the FRED database from the Federal Reserve Bank of St. Louis as well as the database for tangible and intangible capital and investment presented by Bontadini et al. (2021).

Table 1: Average normalized ($y = 1$) values for 2007-2017.

Variable	Value
Aggregate consumption	0.681
Intangible investment	0.127
Tangible investment	0.105
Labour hours	0.343
Commercial debt	0.121
Commercial interest rate	0.029

Other steady-state values were then calculated using the relations described above. I calibrated the persistence and standard shocks by constructing series for A^1, A^2, and m from the data as shown in Figure 1.

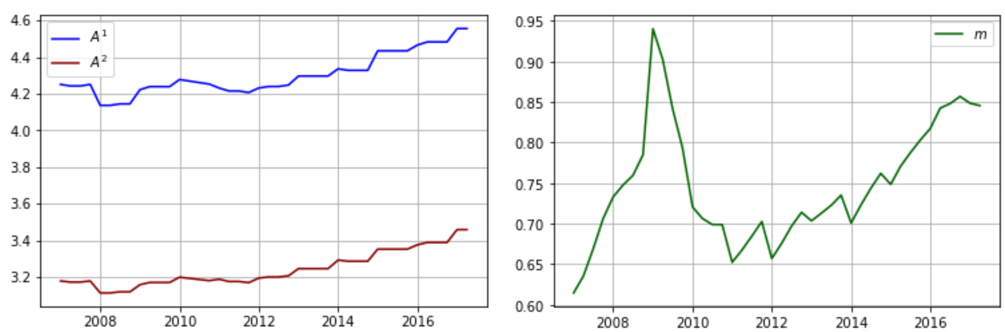


Figure 1: Time series for calculated exogenous variables.

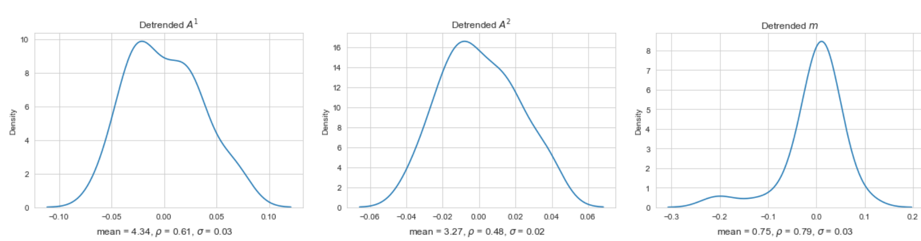


Figure 2: Distributions of deviations from the steady state for exogenous variables A^1, A^2, and m.

The mean value of m is found to be 0.75, in comparison to 0.89 as used for entrepreneurial collateral in Iacoviello (2005) and 0.7 as in Buera, Jaef, and Shin (2015). I follow Iacoviello (2005) and set $\beta = 0.99$ and $\gamma = 0.98$ for the depreciation rates of households and entrepreneurs, respectively and use $\delta_T = 0.033$ for the deprecation rate of tangible capital as per McGrattan and Prescott (2010). Corrado, Haskel, Jona-Lasinio, and Iommi (2022) report that intangible capital does indeed depreciate at varying rates depending on the subset (for example, technology becoming obsolete or marketing efforts fading from relevance). Estimating δ_I is difficult given the low reporting and diversity of intangibles, and I set $\delta_I = \delta_T$ as a benchmark. My results are not sensitive to reasonable changes to the value. Finally, the parameters $\{\psi, \theta, \phi\}$ are calculated as in McGrattan and Prescott (2010) from the steady-state values of the variables and taxes are set as in McGrattan and Prescott (2010), shown in Table 2:

Tax	Value
τ_c	0.066
τ_h	0.311
τ_p	0.35
τ_d	0.15
τ_k	0.0075

Table 2: Values for tax rates.

The value of n is challenging to estimate and likely heavily dependent on the composition of a firm's intangible assets. The ability to document, patent, and repossess intangible assets will effect their collateral value, which will differ wildly between firms: a firm with many patentable intangible assets will face a different constraint than a firm with the majority of its intangibles in, say, organizational efficiency. Amable, Chatelain, and Ralf (2010) propose that 25% of of the value of R&D assets' can be collateralized. I compute results for three cases: $n = 0$ (i.e. intangibles cannot be collateralized), $n = 0.5$, and $n = 1$.

5 Results

5.1 $n = 0$

I conducted simulations with stochastic shocks to A^1, A^2, and m across different values of n, the relative value of intangible capital in the collateral constraint. The impulse responses to a positive shock to m in the case that $n = 0$ are shown in Figure 3. Under the assumption that intangibles cannot be collateralized, output overall rises slightly in response to a credit expansion while investment in both tangibles and intangibles falls. Capital and labour allocated for output production rise at the expense of the share of inputs for intangible production, and total labour increases while total tangible capital falls modestly.

This positive shock to m spurs a small and short-lived increase to entrepreneurial consumption and a persistent decline in household consumption. With an expansion of borrowing ability, the 'savers' (households) forgo more immediate consumption to lend more and 'borrowers' are able to consume at a marginally higher rate. The initial rise in the collateral value of assets raises borrowing, as seen in Figure 4, but the persistent higher interest rate appears to discourage loans and borrowing soon drops below steady-state.

The increase in output causes interest rates to rise according to the formulation of the optimal investment rule. Entrepreneurs then substitute away from accumulating more capital, which would increase the debt load and their interest expense, for greater amounts of labour. As such, capital investment falls in both sectors and labour rises. The reduction in capital reduces the denominator in the interest rate rule which preserves the higher interest rates even after output has returned to steady-state.

Figure 3: Impulse responses to shocks to m_t for $n = 0$.

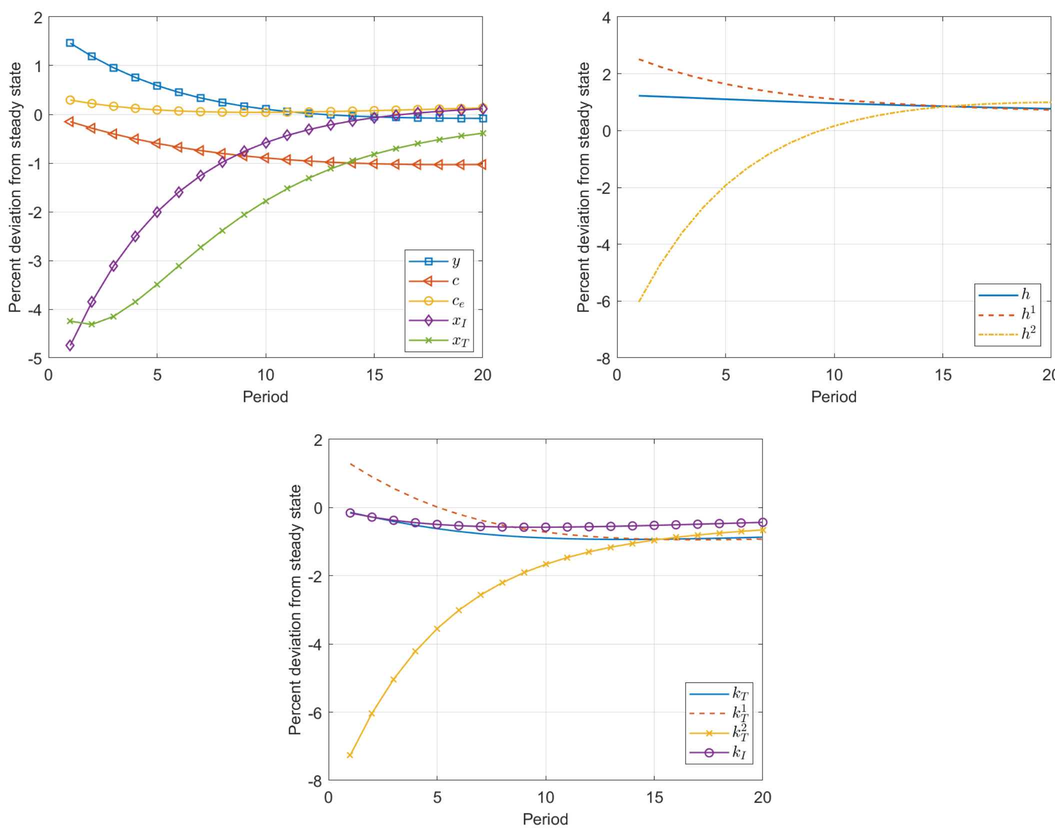


Table 3 displays the correlations between variables for this model. I find an inverse relationship between household and entrepreneurial consumption, with household consumption weakly countercyclical while en-

Figure 4: Impulse responses for credit variables for a positive shock to m_t when $n = 0$.

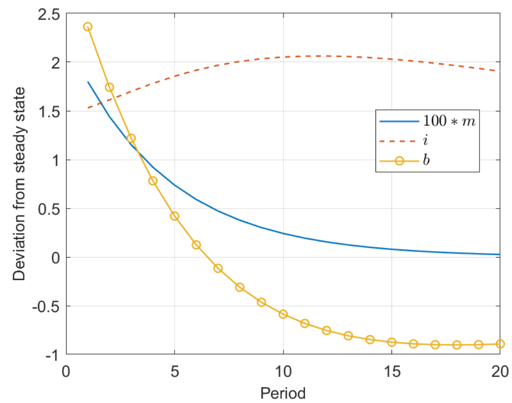


Table 3: Variable correlations for $n = 0$.

	y	c	c_e	x_T	x_I	h	b	i
y	1.000							
c	-0.236	1.000						
c_e	0.207	-0.772	1.000					
x_T	-0.898	0.134	0.105	1.000				
x_I	0.093	0.024	-0.144	-0.212	1.000			
h	0.682	-0.839	0.591	-0.597	0.204	1.000		
b	0.245	0.362	-0.018	-0.048	-0.703	-0.284	1.000	
i	0.537	-0.538	0.248	-0.536	0.656	0.799	-0.628	1.000

trepreneurs consume procyclically. Tangible investment is strongly negatively correlated with output, which is likely driven by the rise in interest rates during shocks to output productivity as seen in Equation (3.10). The spike in interest rates is a burden the entrepreneur's budget, and the entrepreneur chooses to drop tangible investment in response. This is supported by the negative relationship between tangible investment in Table 3 and interest rates, while intangible investment is positively correlated.

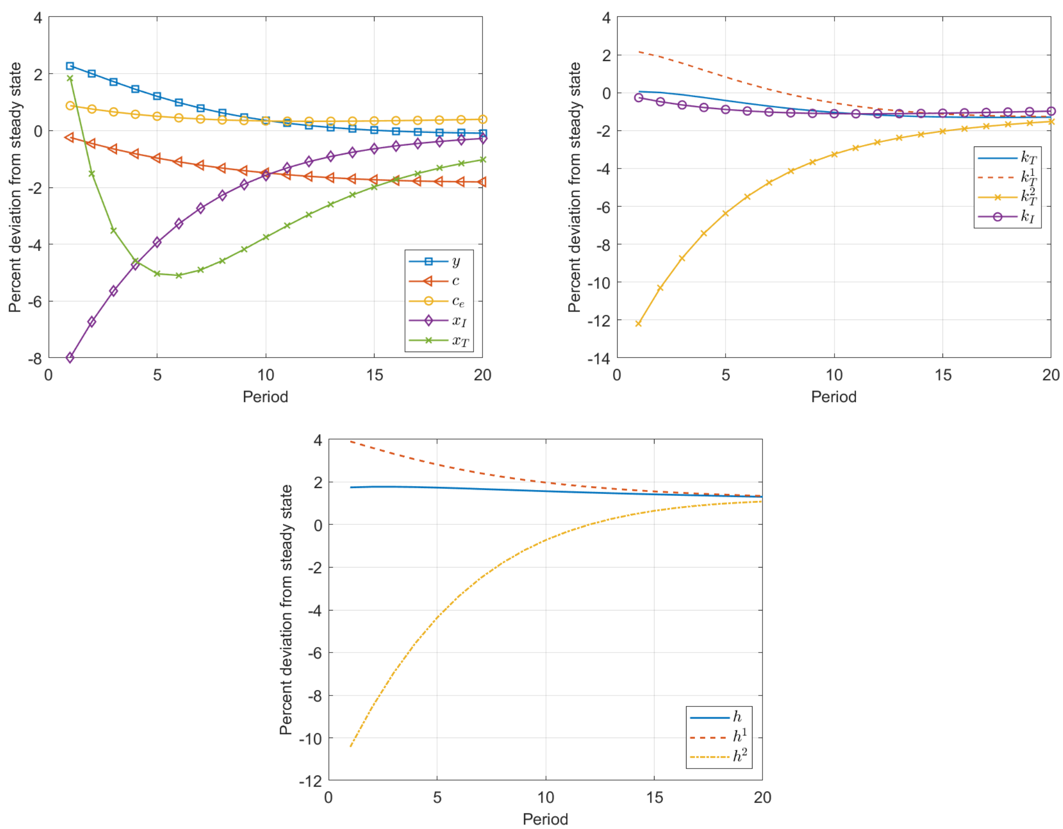
I also observe a seemingly paradoxical inverse relationship between labour provided by the household and household consumption. This is explained by the household's role as the supplier of loans– the household chooses to save more instead of consume when earning more income, generating this relationship between household consumption, debt, and labour hours.

5.2 $n = 0.5$

I now consider the case where intangibles are half as valuable as collateral as tangibles. Most notably, the magnitude of the impulse responses increases and the response of tangible investment x_T becomes positive while intangible investment x_I remains negative (Figure 5). This aligns with the literature wherein larger amounts of available credit act as accelerators and amplify shocks throughout the economy. Variables follow similar paths as in the $n = 0$ case except for the starker drop in capital and labour allocated for intangible production: despite the increased value of intangibles as collateral, the entrepreneur continues to prioritize tangible investment to mitigate the increase in interest rates. Nevertheless, interest rates rise and remain above steady-state as before and the entrepreneur swiftly responds by borrowing less. The reduction in capital keeps the marginal productivity high and so the elevated interest rates endure and more labour is hired to maintain production.

Tangible investment initially rises above steady-state as the entrepreneur's budget expands with the increase in credit. However, the high cost of this debt discourages further capital investment, which then dives below trend and recovers slowly. Output is also eventually hurt by the persistently high interest rates and dips slightly below steady-state due to the falling capital stock. The credit expansion thus becomes a double-edged sword: output initially booms, but is later depressed by high interest rates.

Figure 5: Impulse responses to variables for a positive shock to m_t when $n = 0.5$.



The impacts of interest rates on investment are made obvious in the impulse responses to output produc-

Figure 6: Impulse responses for credit variables for a positive shock to m_t when $n = 0.5$.

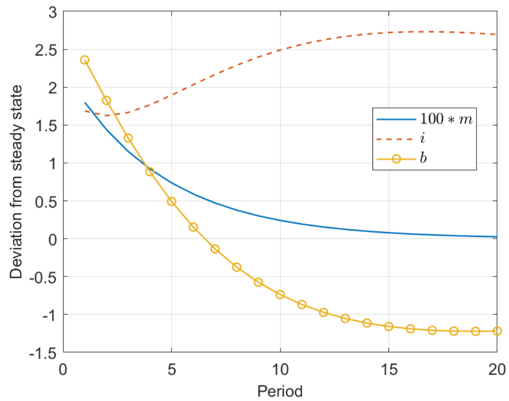
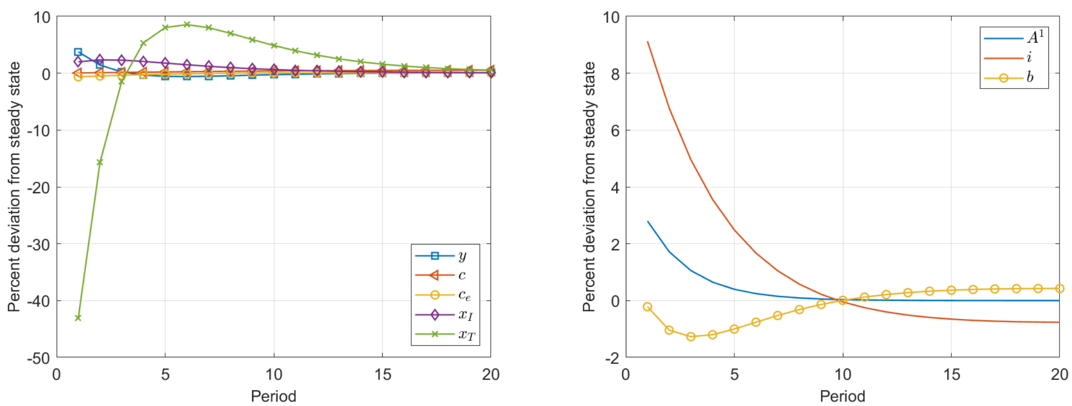


Figure 7: Impulse response functions for shocks to A^1 for aggregate and debt variables.



tivity shocks, shown in Figure 7. The spike in the marginal productivity of capital in output sends interest rates skyrocketing, and the entrepreneur drastically decreases tangible investment in response. A greater share of the existing tangible stock is diverted to producing intangibles, which have no direct influence on interest rates and a weaker impact on the collateral constraint. The return to baseline of productivity and the subsequent settling of interest rates allows tangible investment to surge above steady-state and return capital to its equilibrium value. The interest rate channel induces significant volatility into investment and amplifies the effects of different shocks, as previously found in the literature.

Allowing the collateralization of intangibles in the $n = 0.5$ cases strengthens the relationship between household consumption and borrowing while the correlation between output and interest rates does not significantly change (Table 4). Now that the entrepreneur can pledge more assets as collateral, the household reduces consumption by a larger degree to loan more to the entrepreneur. The model also produces a less negative relationship between intangible investment and debt as compared to when intangibles cannot

Table 4: Variable correlations for $n = 0.5$.

	y	c	c_e	x_T	x_I	h	b	i
y	1.000							
c	-0.474	1.000						
c_e	0.547	-0.915	1.000					
x_T	-0.656	0.121	0.016	1.000				
x_I	-0.265	0.122	-0.230	-0.193	1.000			
h	0.728	-0.929	0.848	-0.421	-0.065	1.000		
b	0.155	0.606	-0.298	0.133	-0.437	-0.492	1.000	
i	0.532	-0.688	0.520	-0.475	0.406	0.819	-0.628	1.000

support debt, as is intuitive. The higher debt limit additionally increases the positive association of debt with the household's consumption, who earn more interest income.

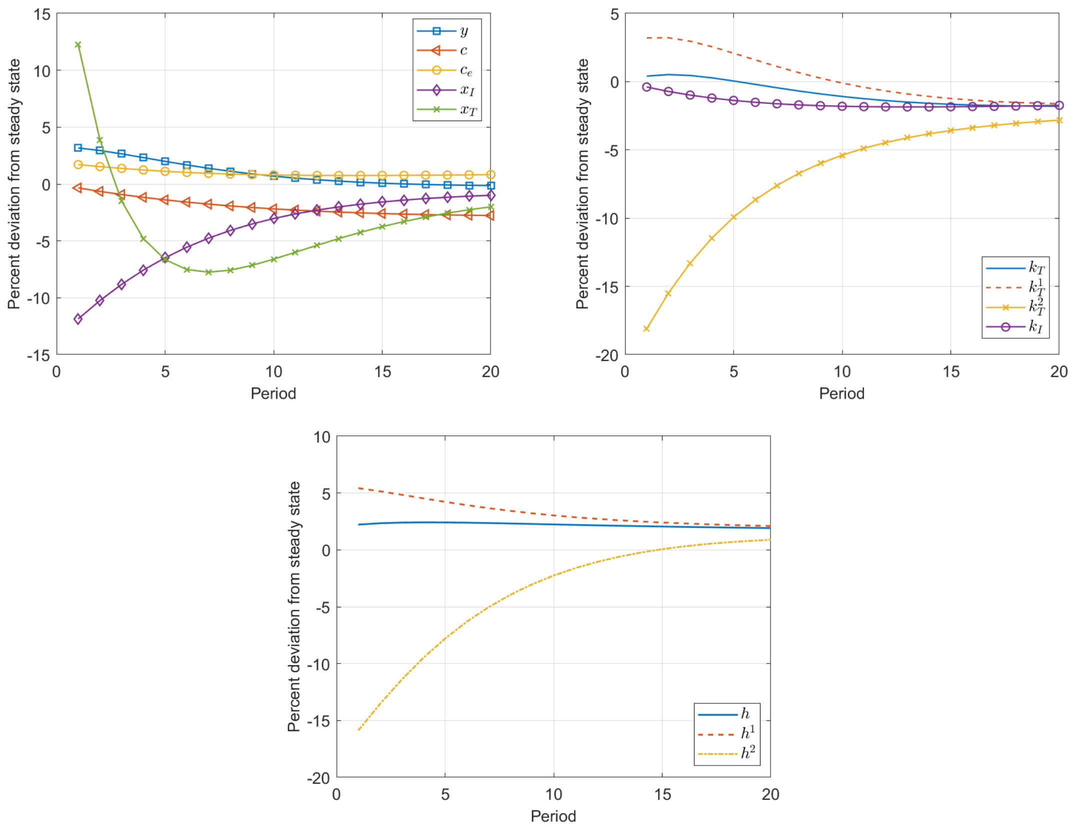
5.3 $n = 1$

Perhaps counterintuitively, the case of $n = 1$ where intangibles are equally valuable as collateral as tangible assets results in a large and strong positive response to tangible investment but a negative response for intangible investment nearly equal in magnitude. Despite the equivalent collateral value of the assets, the entrepreneur chooses to allocate more labour and tangible capital towards output production (Figure 8) instead of intangible production. The expansion of credit is used to fund greater tangible investment at the outset, but the increased interest rates quickly discourage borrowing and investment is cut back to mitigate interest expenditures. Intangibles have no direct effect on interest rates and so can be reduced to balance the entrepreneur's budget without propagating interest effects; on the other hand, accruing more tangible capital in the output sectors reduces its marginal product and helps to soften interest rate increases. The competing effects of the debt burden and the preference for tangibles is reflected in the appearance of a U-shaped curve in the interest rate response in Figure 9, which becomes more pronounced at higher values of n.

Overall, the responses of interest rates and loans (Figure 9) continue the trends from before. Loans initially go up, but then sink below steady-state while interest rates rise and remain above trend due to the reduction in capital.

Once again, raising n results in more interest income for the household and a near-perfect inverse relationship between household and entrepreneurial consumption (Table 5). The correlation between intangible investment and borrowing becomes practically zero as opposed to negative as in previous iterations of the model. High values for the relative collateral value of intangibles are required to break the negative rela-

Figure 8: Impulse responses to variables for a positive shock to m_t when $n = 1$.



tionship between intangible investment and debt. A weak positive correlation between debt and tangible investment appears as expected from the impulse responses now that debt is no longer solely dependent on tangible stock. The model also produces an increasingly negative correlation between output and intangible investment for larger n and a weakening, albeit still negative, correlation between tangible investment and output due to the tightening of the entrepreneur's budget under higher interest rates.

In Tables 6 and 7, I directly compare the models under different n assumptions. As expected from the impulse response functions, output becomes more strongly positively correlated with m at higher values of n (Table 6). Consumption correlation is weakly negative for the household and weakly positive for the entrepreneur with similar strengths across all n values. The correlation between borrowing and m weakens as n increases due to the rising cost of debt, where credit shocks induce more substitution away from capital. The negligible relation between tangible investment and the collateral constraint across all n regimes results from the oscillatory behaviour of tangible investment after credit shocks, where periods of both above- and below-trend investment follow credit expansions. The response of tangible investment is also much stronger

Figure 9: Impulse responses for credit variables for a positive shock to m_t when $n = 1$.

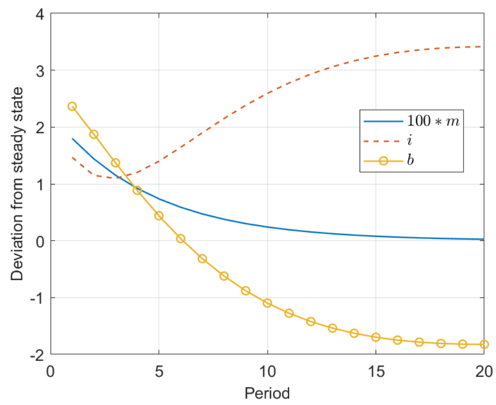


Table 5: Variable correlations for $n = 1$.

	y	c	c_e	x_T	x_I	h	b	i
y	1.000							
c	-0.579	1.000						
c_e	0.674	-0.961	1.000					
x_T	-0.404	0.133	-0.040	1.000				
x_I	-0.580	0.286	-0.358	-0.139	1.000			
h	0.731	-0.969	0.941	-0.307	-0.315	1.000		
b	-0.161	0.836	-0.665	0.238	-0.001	-0.767	1.000	
i	0.497	-0.806	0.701	-0.404	0.100	0.855	-0.778	1.000

for productivity shocks in comparison to credit shocks as seen in Figure 7. On the the other hand, intangible investment becomes more strongly negatively correlated with m as n rises. That is, for greater collateral values of intangible assets, the entrepreneur chooses in invest less in intangibles.

An explanation for this investment behaviour is that allowing the collateralization of intangibles raises the total amount that the entrepreneur is allowed to borrow and expands their budget. The entrepreneur then chooses to recruit more labour and produce more output rather than divert resources towards intangibles. Despite this reduced investment, the overall decline in intangible stock is small as shown in Figure 8 and so the transitory increase in output and entrepreneurial consumption is 'worth' the loss of intangibles, which can be reduced without significant penalty to output. As previously discussed, tangible investment has the added benefit of reducing the marginal productivity of capital and therefore the interest rate, which saves it from a similarly definitive negative response to credit expansions as intangibles.

I observe a rise in the relative standard deviation of consumption for both households and entrepreneurs for increased values of n, shown in Table 7, as well as a more modest increase in relative labour variance. Consumption for the entrepreneur becomes particularly volatile: with interest rates tied to the marginal

Table 6: Simulation correlations between variables and m for varied values of n

	m $(n = 0)$	m $(n = 0.5)$	m $(n = 1)$
y	0.353	0.589	0.674
c	-0.170	-0.214	-0.196
x_T	-0.046	-0.021	0.024
x_I	-0.345	-0.594	-0.757
h	0.276	0.348	0.319
b	0.452	0.383	0.110
i	0.150	0.190	0.163

Table 7: Simulation relative standard deviations of variables.

	$n = 0$	$n = 0.5$	$n = 1$
y	1.000	1.000	1.000
c	1.329	1.769	2.240
x_T	13.936	10.511	7.472
x_I	3.541	3.543	3.542
h	1.469	1.752	2.006
b	1.648	1.189	1.323
i	4.633	4.164	3.659

productivity of tangible capital, carrying more debt under a larger n has the potential to cost more in future periods when an unexpected shock to productivity comes. Likewise, greater variation in labour-hours is necessary to support output and the supply of loanable funds when more debt is allowed. The variance of debt itself does not exhibit a similar increase, suggesting that it is the magnitude of the debt load rather than its variance that drives this volatility. While the variance of intangible capital is largely consistent, tangible investment variance falls for larger n with the reduced reliance of debt-related variables on tangible assets. Interest rate variability is lower for higher n for the same reason.

6 Evaluating model performance

Comparing the model to the data, the model greatly over-predicts the variance of consumption (Table 8) even with wealth- and habit-persistent households. Investment in both tangibles and intangibles is also significantly more volatile in all iterations of the model. On the other hand, the model produces series for borrowing and interest rates with lower relative standard deviations compared to the data. Even with the inclusion of wealth in durable goods, the model still assumes that the supply of debt comes from household saving alone and so shocks to output and therefore wages are amplified through the economy. The interest rate rule is also a source of significant volatility, as seen in Figure 7, where all exogenous shocks induce significant swings in investment by raising the cost of borrowing.

I also find that every quantity except debt is negatively or uncorrelated with m as shown in Table 9. In contrast, the model predicts a positive correlation between m and output and a weak positive relation with labour and interest rates. My results for output are consistent with those of Buera, Jaef, and Shin (2015) wherein the credit limit is positively associated with output. The source of the discrepancy with the data may be that my series for m is calibrated from the ratio of debt to capital assets, where debt may increase in times of economic hardship when firms are in greater need. This additional debt may not come from household savings but instead from other sources or programs and so is not considered by the model. This is supported by the lack of correlation between aggregate consumption c and m, which is calculated from series for industrial loans. Additionally, the fall in the value of capital assets would appear as an 'credit expansion' under this construction, as m would appear to rise if debt is not as quick to respond, particularly for long-term debt that is also not including in this model.

Table 8: Relative standard deviations from real-world data, 2007-2017.

	Relative Std. Dev.
y	1.000
c	0.695
x_T	1.764
x_I	1.456
h	0.093
b	3.073
i	7.616

Table 9: Correlations with m for real-world observations.

	m
y	-0.692
c	-0.053
x_T	-0.513
x_I	-0.568
h	-0.691
b	0.678
i	-0.866

As for the correlations between aggregate quantities, the data reveals strong positive relationships between output and both types of investment (Table 10) whereas the model predicts negative or negligible correlations. Output is indeed positively related to interest rates in alignment with the model, but the procyclicality of observed investment makes the correlations between interest rates and investment also positive. In boom times, firms invest more despite higher costs, suggesting that the returns to investment are perceived as greater than the costs. The model either understates the benefits of investment or overstates the interest rate, or both. In addition, borrowing and interest rates are negatively correlated in both the model and the data which supports the idea that high rates do discourage debt– in this case, it appears that firms turn to alternate financing options to fund investment when interest rates are high, such as saving, issuing equity, or employee compensation contracts (Corrado, Haskel, and Jona-Lasinio, 2019; Sun and Xiaolan, 2019; Favara, Gao, and Giannetti, 2021).

Table 10: Correlations between variables for real-world observations.

	y	c	x_T	x_I	h	b	i
y	1.000						
c	0.087	1.000					
x_T	0.852	0.163	1.000				
x_I	0.785	0.129	0.886	1.000			
h	0.706	0.448	0.751	0.744	1.000		
b	-0.082	0.194	0.260	0.124	-0.100	1.000	
i	0.743	-0.069	0.562	0.525	0.659	-0.554	1.000

7 Conclusion

I present a model combining the intangible capital structure and non-neutral technology of McGrattan and Prescott (2010) with a time-varying collateral constraint and durable goods stock in the style of Iacoviello (2005) and Buera, Jaef, and Shin (2015). As a greater fraction of intangible capital can be collateralized, the overall debt available to borrow increases and I observe an increasing positive relationship between the collateral constraint and output while intangible investment becomes more strongly negative. Interest rates, determined by an optimal investment rule, drive significant variability in tangible investment and credit expansions make the firm substitute labour for capital to mitigate the persistently high cost of debt. The decline in intangible investment under higher collateralizability assumptions results from high interest rates, where increased intangible capital increases the debt load with no direct effect on interest rates. Credit expansions also raise borrower consumption and suppress that of lenders, who must forgo consumption to meet the demand for loans.

Overall, the model does not accurately reproduce the observed relationships and behaviours of macroeconomic quantities. Possible reasons include my formulation of the interest rate as dependent on tangible productivity alone which leads to under-estimation of interest rate volatility and over-reliance of the interest rate on output productivity shocks. Additionally, the formulation of durable wealth d and d_e requires revisiting to better incorporate these variables into the model and address the large variation in consumption for both households and entrepreneurs. Future work should explore alternate formulations of the interest rate rule in order to appropriately model variation in investment and debt which the model over- and underpredicts, respectively. A more rigorous definition and calibration of m may also be necessary to isolate the effects of credit availability on aggregate variables rather than relying on surface-level correlations.

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