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Finesse Measurement in Fabry-Perot Interferometers

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Abstract

STAR (Space-time Asymmetry Research) is an experimental satellite that will measure anisotropy in the speed of light to unprecedented resolutions using two orthogonal Fabry- Perot interferometers. In order to achieve the desired level of precision, the optical cavities must have finesse values greater than 100,000. Using two test interferometers, I tested and optimized two different experimental methods for measurement of finesse: full-width half max measurement and cavity ring-down spectroscopy. Although each technique proved valid in its own right, my work suggests that cavity ring-down spectroscopy may be more applicable for the needs of STAR.

Cover Page Footnote

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1 Abstract

STAR (Space-time Asymmetry Research) is an experimental satellite that will measure anisotropy in the speed of light to unprecedented resolutions using two orthogonal Fabry-Perot interferometers. In order to achieve the desired level of precision, the optical cavities must have finesse values greater than 100,000. Using two test interferometers, I tested and optimized two different experimental methods for measurement of finesse: full-width half max measurement and cavity ring-down spectroscopy. Although each technique proved valid in its own right, my work suggests that cavity ring-down spectroscopy may be more applicable for the needs of STAR.

2 Introduction

STAR is the latest in a series of increasingly high-precision reincarnations of the Michelson-Morely and Kennedy-Thorndike experiments, which used interferometers to prove the isotropy of the speed of light. While the original experiments laid the groundwork for special relativity, STAR and its recent predecessors seek to accomplish the opposite. Emerging theories in physics such as quantum gravity suggest minute anisotropy in the speed of light, in direct violation of special relativity. In order to resolve the anisotropy, however, the experiments must be performed with unprecedented precision. Such effects as seismic fluctuations and stress on the experimental setup due to its own weight have provided enough noise to drown out any possible anisotropy in previous experiments. In order to escape these sources of noise, STAR will be the first iteration of the Michelson-Morely and Kennedy-Thorndike ex-

periments to run in space. Yet absence of noise alone is not enough to achieve the resolution required. The interferometers used must be of very high quality. In particular, they must have finesse values of over 100,000 in order to obtain the $3*10^{-18}m/s$ resolution that STAR is aiming for [3].

The interferometers to be used in STAR will be of a variety called *Fabry-Perot interferometers*. The Fabry-Perot interferometer, also known as the etalon, consists of two reflective optical flats oriented in such a way as the reflective surfaces face towards each other. The device is distinguished by the dependence of the transmission percentage of light through the cavity on the wavelength of the incident light. Since a photon is much more likely to be reflected back from the second flat towards the first reflecting flat than pass through it, photons essentially become trapped in the cavity. In the case of most wavelengths of light, a photon will undergo a phase shift for each cycle of reflection. After each subsequent cycle, this phase shift is compounded, resulting in a spread of photons evenly out of phase. These photons destructively interfere with eachother and do not contribute to the output of the etalon. For photons with wavelength very close to an integer fraction of the length of the cavity, however, there is sufficiently low phase shift that instead of canceling each other out, each subsequent cycle of photon reflection constructively interferes with the incoming photons, resulting in a standing wave.

As the reflectivity of the optical flats is raised, so too does the amplitude of the standing wave increase; as such, the decrease in probability of transmission of an individual photon is counteracted by an increase in the number of photons trapped in the cavity. So long as the

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light in the cavity remains in phase with itself following internal reflection, the intensity of the intensity of the light being ejected from the cavity is not appreciably negatively correlated with the internal reflectivity of the optical flats [1]. In order for the light in the cavity to form a standing wave, and thus be transmitted through the cavity, the length of the cavity must be an integer multiple of the wavelength of the light. As such, a graph of the transmission percentage versus the wavelength of the incident light will yield "transmission peaks" with periodic spacing[1]. The spacing between the peaks is an inherent property of the specific etalon and is given by a constant known as the free spectral range ($\Delta\lambda$). The more reflective the optical flats in the interferometer, the greater the importance that the incident light be precisely tuned to the cavity. Since higher reflectivities mean that each photon travels further before exiting the cavity, even a slight phase shift occurring with each reflection can build up over time and lead to destructive interference. As such, as the reflectivity of the optical flats increases, the selectivity of the interferometer increases (see figure 1). An increase in selectivity manifests as a narrowing of the transmission peaks of the device. The width of the transmission peak can be characterized by the "full-width half max", the range in λ that corresponds to the region of the peak for which transmission is at least fifty percent of the transmission at the apex, denoted by $\delta\lambda$. We can use $\delta\lambda$ in conjunction with $\Delta\lambda$ in order to write an expression for the selectivity of a particular cavity in terms of a dimensionless quantity known as the "finesse".

$$F = \Delta\lambda/\delta\lambda \tag{1}$$

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[1]This also implies that the transmission peaks will have periodic spacing with respect to frequency

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The finesse of a Fabry-Perot interferometer is not just a measurement of the selectivity of the cavity. As mentioned earlier, a higher selectivity of transmission (and thus higher finesse) occurs due to a higher average number of times a photon is reflected back and forth within the cavity. Mathematically, this means we can also express the finesse in terms of the mean reflectivity R of the optical flats.

$$F = \pi\sqrt{R}/(1 - R) \tag{2}$$

3 Methodology

3.1 Experimental Setup

Our setup, at its core, consisted of a 1064 nm Nd:YAG laser, a platform for the optical cavity, a photodetector to measure the amount of light passing through the cavity, auxiliary optical components to stabilize the frequency of the laser and aid with alignment, and a digital oscilloscope to measure and record the data from the photodectors (see figure 2). The front panel of the laser had two BNC ports for frequency modulation of which one was used in order to implement the Pound-Drever-Hall technique to lock the frequency of the laser to the optical cavity. The experiment was performed in a black box in a dark room, and the photodetector current from ambient light was measured and subtracted from the photodetector's final current measurement. Because the laser was infrared, we introduced an auxiliary 633 nm He:Ne laser to our beam, which allowed us to see the beam and thus

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helped us with alignment. We were provided with two test cavities to work with, labeled "A" and "B", with respective $\Delta\lambda$ values of $1.533 * 10^9$ Hz and $1.636 * 10^9$ Hz. Although their exact finesse values were not known (hence the need for testing finesse measurement techniques), Cavity A was estimated to have a finesse of 10000, and Cavity B was estimated to have a finesse of 30000.

3.2 $\delta\lambda$ Measurement

In order to measure the transmission percentage through the cavity with respect to the wavelength of light (and thus characterize the transmission peak) we fed a trangle wave from a function generator into our laser's fast frequency modulation (FFM) input, allowing us to "sweep" the frequency of our laser back and forth around the optimal frequency for transmission (see figure 3). We recorded the voltage input to the FFM input and the photodetector output at each data point. Using the FFM input, we were able to construct a plot of laser frequency versus transmission for each experimental trial. We used our oscilloscope's built-in full-width half max measurement software to glean δf, and then converted to $\delta\lambda$ using the relation between the two parameters.

$$\delta\lambda = \lambda_0^2 \delta f/c \tag{3}$$

Our test interferometers had previously measured values for $\Delta\lambda$, so we were able to determine the finesse using of equation (1).

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3.3   Cavity Ring-Down Spectroscopy

Cavity ring-down spectroscopy consists of determining the mean internal reflectivity R of the lenses in an optical cavity by very quickly shutto the light source and then measuring the time constant τ of the rate of decay of light from the cavity. The decay rate of light intensity from the cavity takes the form of exponential decay as such:

$$I(t) = I_0 e^{-t/\tau} \tag{4}$$

The relationship between R and τ is given by

$$\tau = \frac{n}{c} \frac{l}{1 - R + X} \tag{5}$$

for n is the index of refraction within the cavity (in our case $n = 1$), c is the speed of light, l is the length of the cavity and X is a constant accounting for optical losses due to imperfections in the setup and photon absorption. In order to yield a usable decay curve, it was of paramount importance that the light shutoff time be significantly lower than the time constant of the decay curve. In order to achieve the requisite shutoff time, we introduced an acousto-optical modulator (AOM) to the experimental setup. When on, the AOM diffracted the path of the laser by a fraction of a degree. We re-aligned the equipment downstream of the AOM to this new path. By turning off the AOM, we were able to divert the laser path to a beam dump and cut off the light incident upon the cavity. Our AOM had a shutoff time of roughly 200 ns, and a high-finesse cavity has a decay time constant on the order of a few microseconds.

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4 Results

We measured $\delta\lambda$ for both cavities and varied a myriad of experimental parameters, including sweep frequency, sweep voltage, and the intensity of the light incident upon the cavity. Of these, the only parameter that had an appreciable effect on the measured finesse was the sweep frequency. In cavity A, we saw possible hints of a weak negative relationship between the sweep frequency and the measured finesse (see figure 4), but for the higher finesse cavity B the negative relationship dominated the data (see figure 5). The average measured finesse of cavity A using the $\delta\lambda$ technique was 12343, which was considered reasonable given that the finesse was estimated to be on the order of 10000. For cavity B, the relationship between the finesse and sweep frequency was such that to give a single value of the finesse would be inappropriate. That being said, cavity B was predicted to have a finesse of approximately 30000 and the measured finesse for very low sweep frequencies was considered within an acceptable range. We were unable to run tests for sweep frequencies lower than 0.1 Hz because that was the lower frequency limit of our function generator.

The decay curve for cavity ring down spectroscopy can be dominated for lower finesse cavities by the finite shutoff time of the light source. As a result, cavity ring down spectroscopy was only successfully tested in cavity B. However, the measured finesse ranged from 27577 to 32623, with a standard deviation of 1390 and mean of 30218 (see figure 6), which was considered within the acceptable range.

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5   Discussion

The STAR satellite will require Fabry-Perot interferometers with finesse values greater than 100000 [3]. $\delta\lambda$ measurement appears to be appropriate for a lower-finesse cavity, but even at $F = 30000$, the data is almost unusable because of the dominating effect of the sweep frequency. We would expect the sweep frequency to play even more of a role at $F > 100000$, which would be unacceptable for STAR's purposes. It may be possible to eliminate the effects of the sweep frequency variation on the finesse, however. Zooming in on the transmission peak, we saw a horizontal stretching of the falling edge of the peak that correlated with the decrease in finesse measurements at higher sweep frequencies. This stretching certainly led to increased full width half measurements, resulting in deflated finesse. Since the stretching only appears to occur on the falling edge of the peak, by manually measuring the width of the rising edge at half-max, doubling it, and then using that as $\delta\lambda$ we may be able to fix the issue. We were unable to test this hypothesis, however, because our oscilloscope's resolution was too low to manually determine the half width half max. It is likely that the horizontal stretching is an artifact of RC effects disrupting the oscilloscope's signal over short timescales. Since we were using the rather weak raw output of the photodetector for our signal, we were forced to use the $1M\Omega$ coupling for the input of our oscilloscope (we could not resolve the transmission peaks at all for the 50Ω coupling). Even though the capacitance of our BNC connector cable was minute (on the order of picofarads), it is possible that the cable's discharge time was long enough that a discharge decay curve would be superimposed on our transmission peak, resulting in an artificial stretching of the falling edge. This issue

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is difficult to address, however. In order to switch to 50Ω coupling we would need to amplify the signal from the photodetector, which introduces even more instances in which stray capacitance could wreak havoc with the signal. Cavity ring-down spectroscopy, by contrast, yielded a relatively straightforward picture of the finesse. The data was in accordance with our prediction for cavity B, and although we were unable to glean a result for a lower finesse cavity, any cavity that might possibly be used in the STAR satellite will have a finesse value much greater than any cavity we measured. Additionally, a higher finesse cavity would yield a slower decay, meaning that the effects of light shut-off would be further mitigated.

6   Conclusion

Our results suggest that the finesse of Fabry-Perot interferometers is best measured by $\delta\lambda$ measurement for a low-finesse cavity, but that cavity ring-down spectroscopy becomes more appropriate somewhere between $F = 10000$ and $F = 30000$, and remains preferable as the finesse further increases. Although it may not be impossible to use $\delta\lambda$ measurement to experimentally measure finesse values over 100000, our data suggests that cavity ring down spectroscopy provides an easier, more accurate way to do so.

7   Acknowledgments

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and built the initial setup for the experiment, and to Stanford University for providing the

laboratory and funding for the project.

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8 Figure Captions

Figure 1. Graph demonstrating the relationship between finesse/selectivity and the character of the transmission peak.

Figure 2. Optical diagram with key

Figure 3. Screenshot showing oscilloscope interface for $\delta\lambda$ measurement. The smaller peaks on each side of the transmission peak are an artifact of the PDH locking system we used to stabilize the laser's frequency and are not part of the data.

Figure 4. XY plot of sweep frequency vs measured finesse for Cavity A showing possible weak negative correlation between the two parameters

Figure 5. XY plot of sweep frequency vs measured finesse for Cavity B showing over-powering dependence of finesse on the sweep frequency

Figure 6. Chart of measured time constants and corresponding calculated finesse values for cavity ring-down spectroscopy
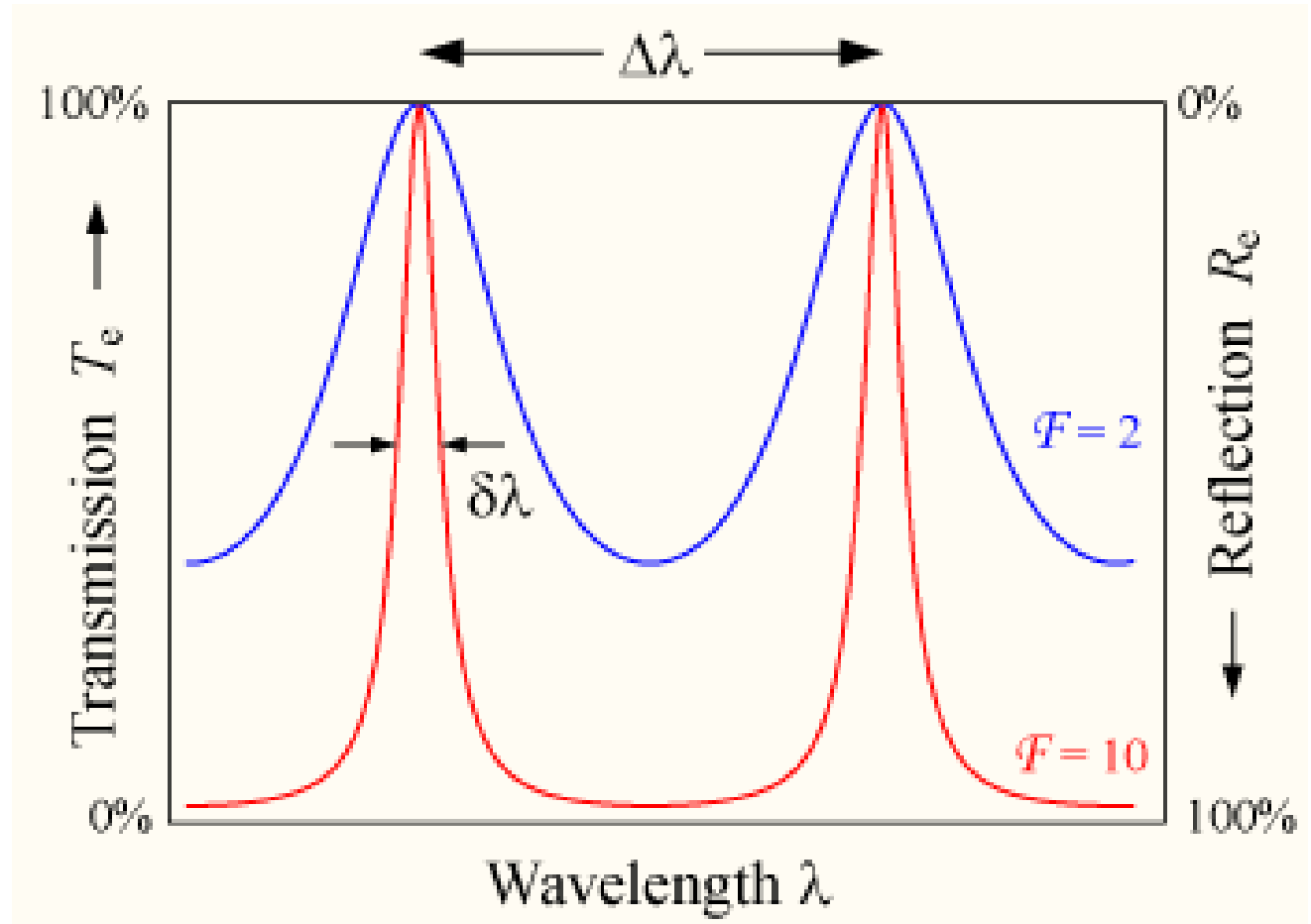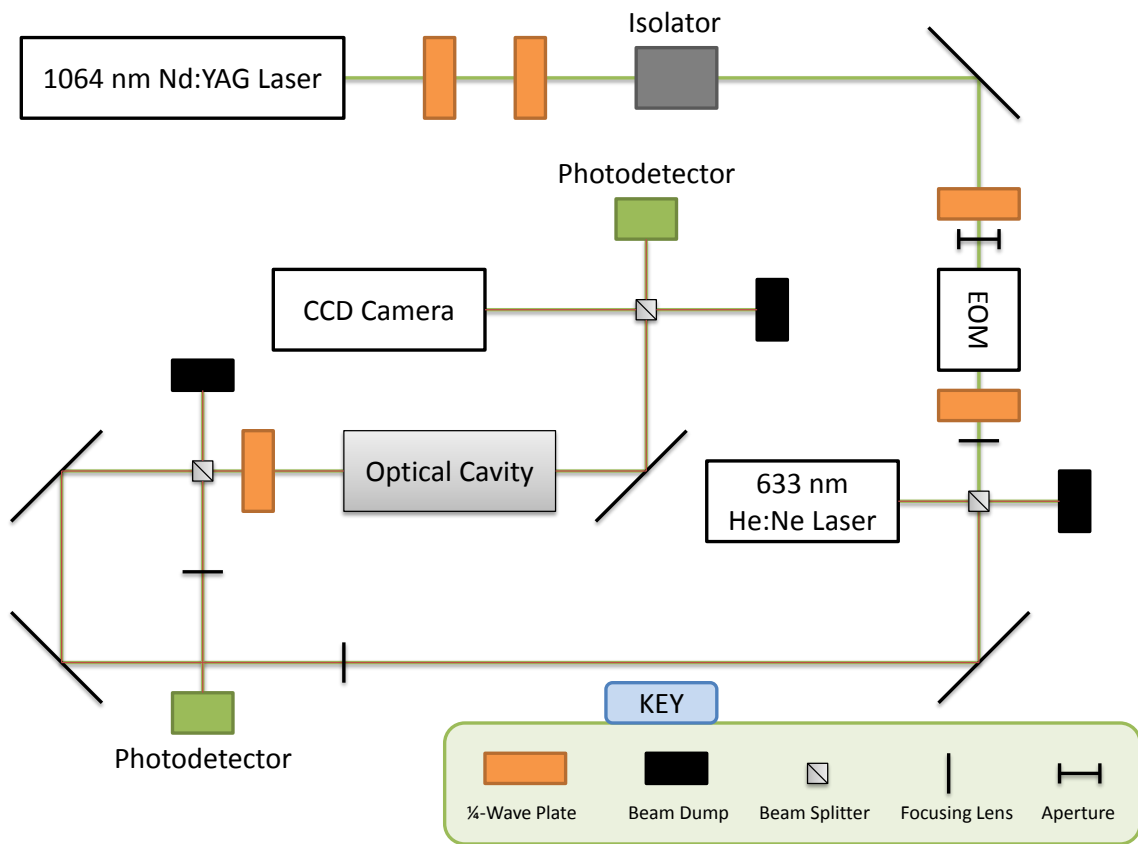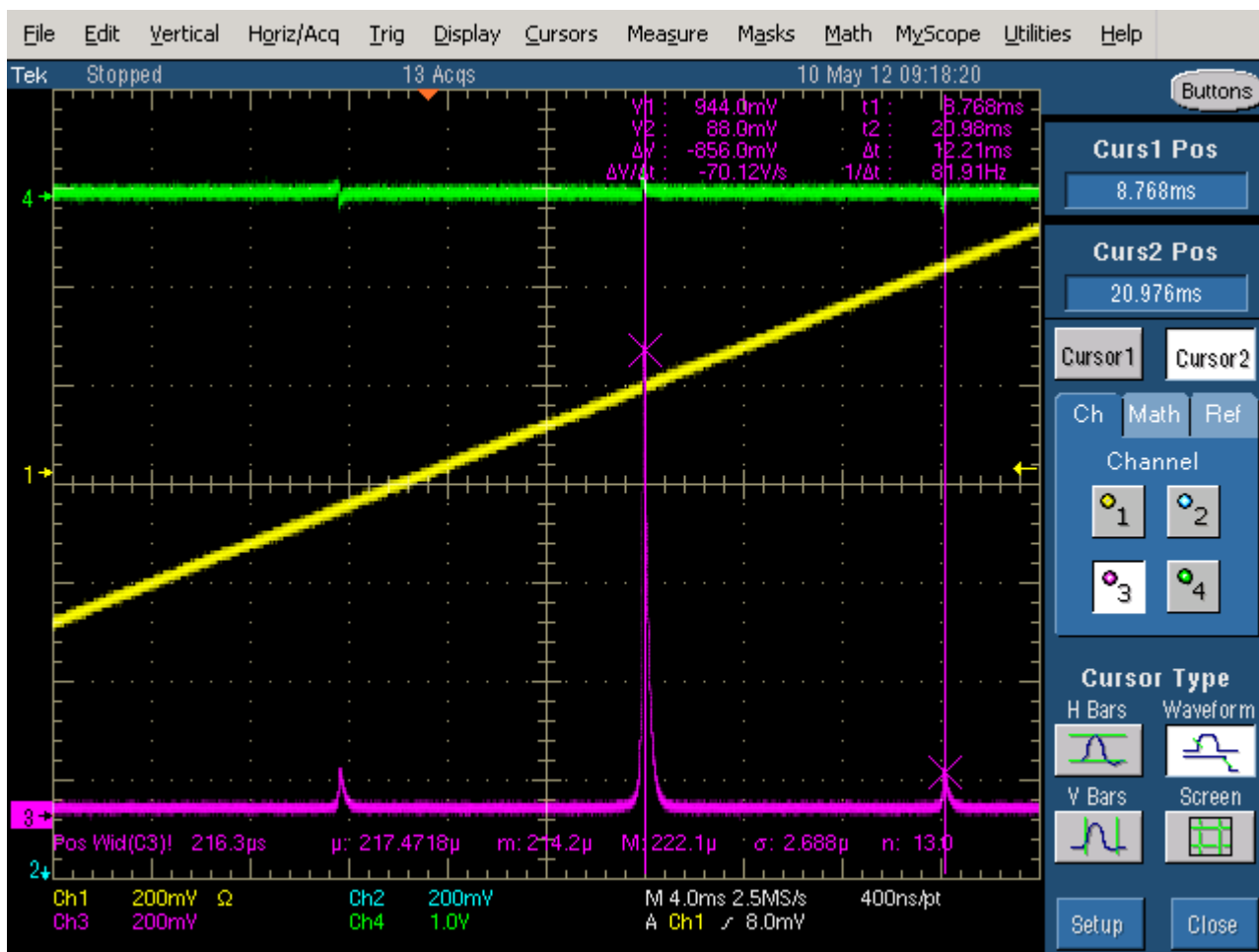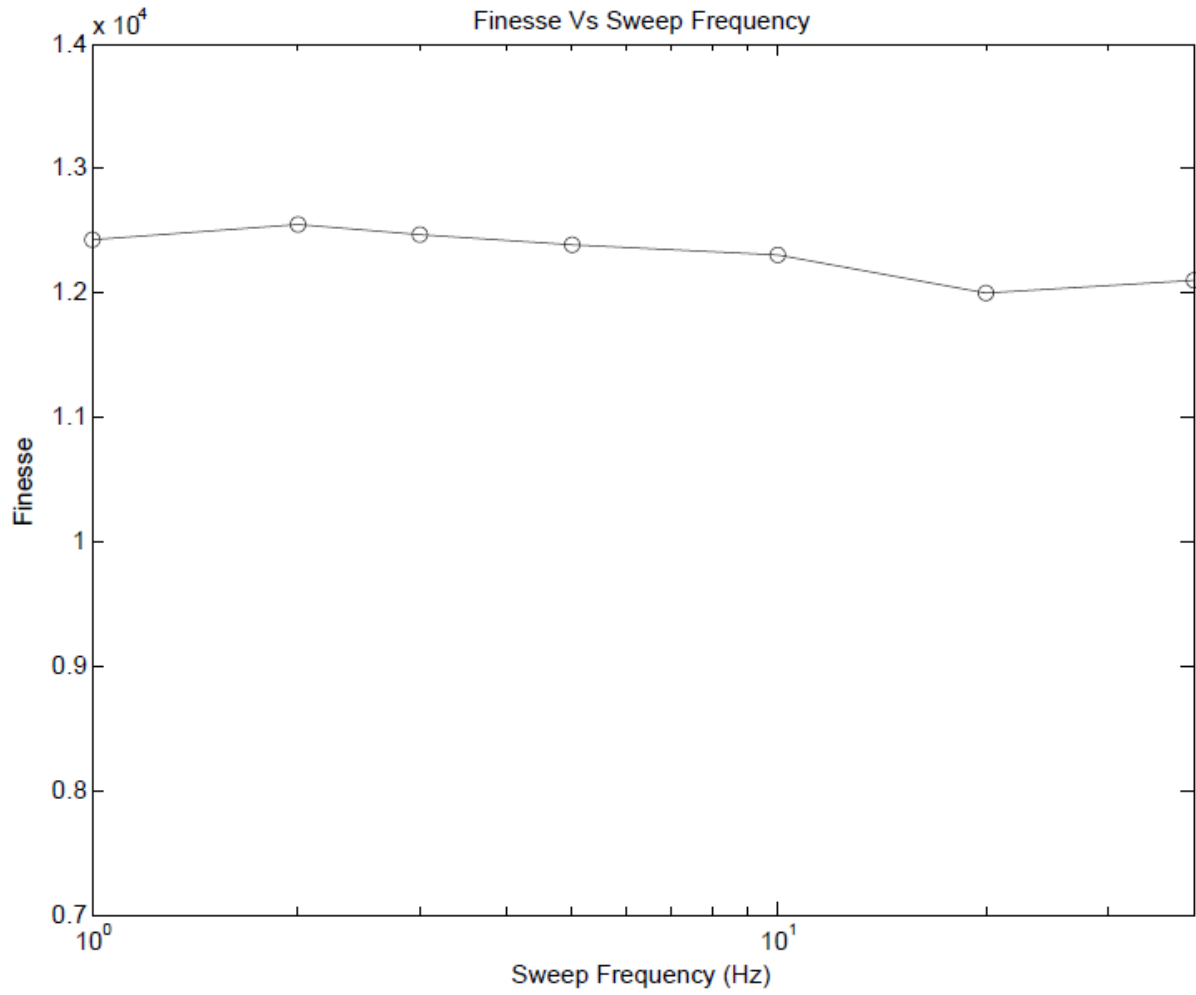
9 Figures

Figure 1

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Figure 2

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Figure 3

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Figure 4

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Figure 5

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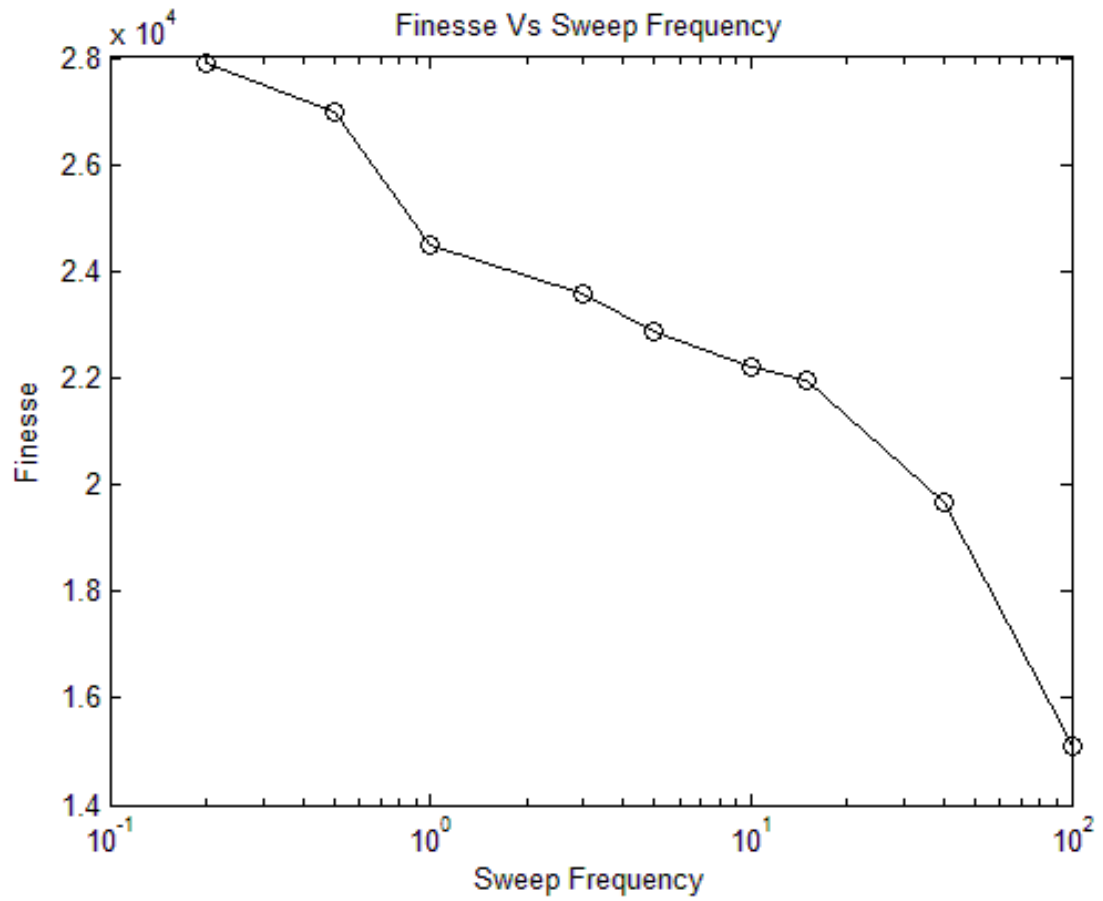
Measurement Number	Time Constant (microseconds)	Finesse
1	3.19	30720
2	3.13	30180
3	3.39	32623
4	3.06	29487
5	3.27	31523
6	2.88	27727
7	3.16	30461
8	3.16	30406
9	3.19	30724
10	3.19	30747
11	3.16	30420
12	3.32	32036
13	3.33	31220
14	3.06	29474
15	2.86	27595
16	2.89	27790
17	3.31	31898
18	3.03	29239
19	3.14	30259
20	3.09	29823

Figure 6

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